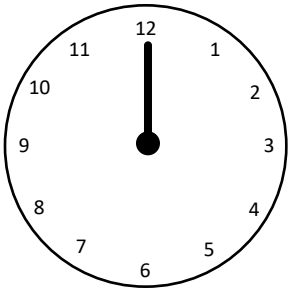
Crash Course on Modular Arithmetic

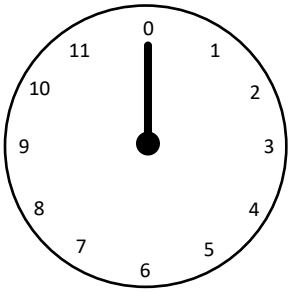
The Clocks

It is now 12 o'clock. What number is the hour hand on in:



- 9 hours?
- 12 hours?
- 16 hours?
- 24 hours?
- 35 hours?

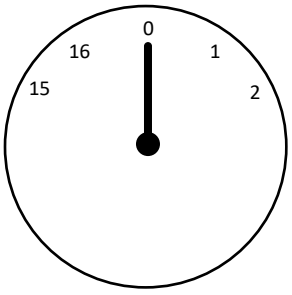
We now tweak the clock (replace 12 with 0). It is now 0 o'clock. What number is the hour hand on in:



- 9 hours?
- 12 hours?
- 16 hours?
- 24 hours?
- 35 hours?
- 71 hours?

We now tweak the clock again – it now has 17 hours.

- Optional: Fill in the rest of the clock.



It is now 0 o'clock. What number is the hour hand on in:

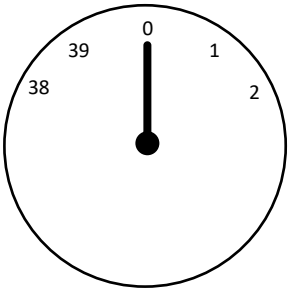
- 9 hours?
- 12 hours?
- 16 hours?
- 24 hours?
- 35 hours?
- 71 hours?

Fill in the table:

	Remainder	…and we say:
$9 \div 12$	9	$9 \equiv 9 \pmod{12}$
$12 \div 12$	0	$12 \equiv 0 \pmod{12}$
$16 \div 12$		$16 \equiv \quad \pmod{12}$
$24 \div 12$		$24 \equiv \quad \pmod{12}$
$35 \div 12$		$35 \equiv \quad \pmod{12}$
$71 \div 12$		$71 \equiv \quad \pmod{12}$
$9 \div 17$	9	$9 \equiv 9 \pmod{17}$
$12 \div 17$		$12 \equiv \quad \pmod{17}$
$16 \div 17$		$16 \equiv \quad \pmod{17}$
$24 \div 17$		$24 \equiv \quad \pmod{17}$
$35 \div 17$		$35 \equiv \quad \pmod{17}$
$71 \div 17$		$71 \equiv \quad \pmod{17}$

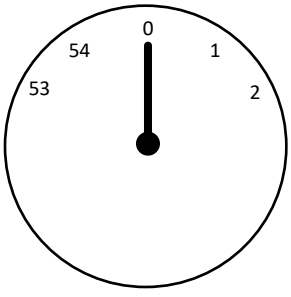
Now harder:

- $7 \equiv \quad \pmod{40}$
- $23 \equiv \quad \pmod{40}$
- $161 \equiv \quad \pmod{40}$



Even harder:

- $8 \equiv \quad \pmod{55}$
- $64 \equiv \quad \pmod{55}$
- $8^3 = 512 \equiv \quad \pmod{55}$
- $8^4 = 4096 \equiv \quad \pmod{55}$
- ⋮
- $8^7 = 2097152 \equiv \quad \pmod{55}$



What about:

- $2^{23} = 8388608 \equiv \quad \pmod{55}$